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APPROXIMATE CALCULATION OF PURE BENDING OF A SYMMETRIC BEAM UNDER IRREVERSIBLE STRAIN

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The method of characteristic parameters is extended to the case of creep bending of a beam whose cross section has one symmetry axis. The method proposed is very time consuming and based on simple mathematical apparatus. The problem reduces to solving simple nonlinear algebraic equations.

In aircraft and ship building, the basic parts are produced by forming processes involving slow deformation, in which the irreversible strains are considerably contributed by creep strains [1]. In this case, lower forming loads are required and the accuracy in manufacturing parts increases. One of the main technological problems is to determine the shape of a member after the load is removed. Direct calculations are very involved because of the complex geometry of the members and the nonlinearity of the equations.

The calculations are simplified using the method of so-called characteristic parameters [2, 3]. The essence of the method is the following. It is assumed that in a loaded structural member there is a certain characteristic point (neighborhood), whose position depends only slightly on the load, strain rate, and temperature under specified boundary conditions. For a constant load, the stresses at this point remain unchanged and equal to the elastic stresses up to the moment the member fails. If the external load is varied, the stresses at the characteristic point vary by the law of uniaxial elastic deformation and the strains at this point determine the behavior of the entire structure. After the load is removed, the stresses at this point vanish. The coordinate of the characteristic point is determined as the point at which elastic and steady stress diagrams intersect. In [2, 3], the coordinates of the characteristic point were found and used to analyze the creep bending of a rectangular beam. In the present paper, the method is extended to a beam whose cross section possesses one symmetry axis (for example, T-shaped cross sections with one or more steps).

We consider a beam loaded by a bending moment M which is constant along the beam. It is assumed that the beam obeys the Kirchhoff hypothesis of direct normals:

$$\sigma/E + \varepsilon^c = \chi(z - \delta).$$

Here z is the coordinate reckoned along the height of the beam ($z = 0$ at the base of the beam), σ is the stress at the point z, E is Young's modulus, ε^c is the creep strain, χ is the beam curvature, and δ is the coordinate of the neutral plane (total strain vanishes in this plane).

We write the equilibrium equations

$$\int_S \sigma\, dS = 0, \qquad \int_S \sigma z\, dS = M, \tag{1}$$

where S is the cross-sectional area of the beam.

At the initial moment $t = 0$, we have $\varepsilon^c = 0$. Therefore,

$$\sigma = E\chi(z - \delta_0), \tag{2}$$

where δ_0 is the initial position of the neutral plane.

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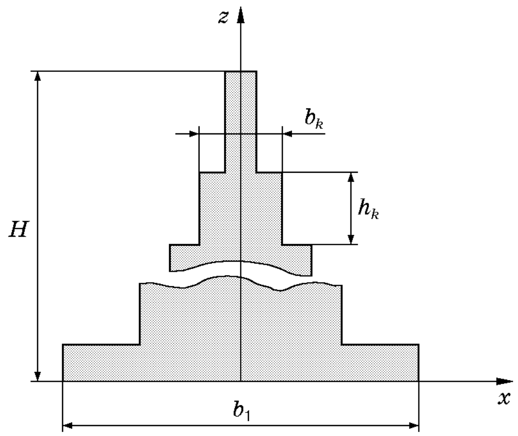

Fig. 1

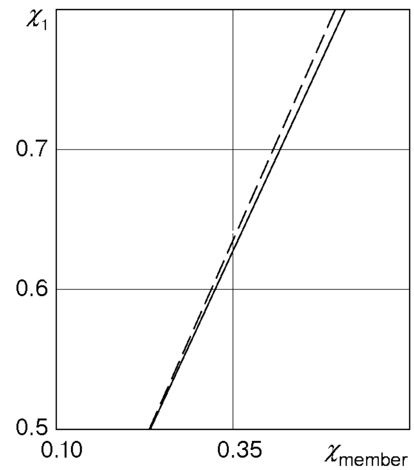

Fig. 2

From the first equation in (1), we obtain

$$\delta_0 = \frac{1}{S}\int\limits_0^H b(z)z\,dz, \tag{3}$$

where H is the height of the beam and $b(z)$ is its width at the point z.

We use the creep law

$$\dot{\varepsilon}^c = B\sigma|\sigma|^{n-1}, \tag{4}$$

where B and n are material's constants.

Thus, the steady-stress distribution in the beam is given by

$$\sigma = (\dot{\chi}/B)^{1/n}(z-\delta)^{1/n}. \tag{5}$$

At the characteristic point $z = \hat{z}$, the stresses defined by (2) and (5) are equal. Hence,

$$\hat{\sigma} = E\chi(\hat{z}-\delta_0) = (\dot{\chi}/B)^{1/n}(\hat{z}-\delta)^{1/n} = (M/J)(\hat{z}-\delta_0), \tag{6}$$

where $J = \int\limits_0^H b(z-\delta_0)z\,dz$ is the elastic moment of inertia of the beam.

Using the first equation of (1), we obtain the following equation for the steady location of the neutral plane δ

$$\int\limits_0^H b(z)(z-\delta)^{1/n}\,dz = 0. \tag{7}$$

Since the beam moment is constant, the second equilibrium equation in (1) and relation (6) can be combined to give

$$(\hat{z}-\delta)^{1/n} = \int\limits_0^H b(z)(z-\delta)^{1/n}z\,dz\biggl(\int\limits_0^H b(z)z^2\,dz - S\delta_0^2\biggr)^{-1}(\hat{z}-\delta_0). \tag{8}$$

Solving the nonlinear equations (7) and (8) with allowance for (3), we determine the coordinate of the characteristic point $\hat{z}$, which depends only on the creep exponent n and the geometrical dimensions of the beam. Given the bending moment and the rate of change in beam curvature, one can determine the stress $\hat{\sigma}$ at the point $\hat{z}$.

Let us consider a beam of multistep cross section which is symmetrical about the vertical axis (Fig. 1). Using (3), (7), and (8), we obtain

$$\delta_0 = \sum_{k=1}^m b_k(h_k^2 - h_{k-1}^2)\Bigl(2\sum_{k=1}^m b_k h_k\Bigr)^{-1}, \quad \sum_{k=1}^m b_k\Bigl((h_k-\delta)^{(n+1)/n} - (h_{k-1}-\delta)^{(n+1)/n}\Bigr) = 0,$$

$$(\hat{z}-\delta)^{1/n} = \frac{n}{n+1}\sum_{k=1}^{m} b_k\Big[h_k(h_k-\delta)^{(n+1)/n} - h_{k-1}(h_{k-1}-\delta)^{(n+1)/n} - \frac{n}{2n+1}((h_k-\delta)^{(2n+1)/n} - (h_{k-1}-\delta)^{(2n+1)/n})\Big]$$
$$\times\Big[\frac{1}{3}\sum_{k=1}^{m} b_k(h_k^3-h_{k-1}^3) - \Big(\sum_{k=1}^{m} b_k(h_k^2-h_{k-1}^2)\Big)^2\Big(4\sum_{k=1}^{m} b_k h_k\Big)^{-1}\Big]^{-1}(\hat{z}-\delta_0).$$

Here $h_0 = 0$, b_k and h_k are the width and height of the kth step, respectively, and m is the number of steps.

The beam is bent by a constant moment M. The creep law is taken in the form (4). We assume that plastic strains are absent. Hence, at the initial moment $t = 0$, the curvature of the beam is given by $\chi_0 = M/(EJ)$. For $t = t_1$, the total curvature of the beam is written as $\chi_1 = \chi_0 + \Delta\chi$.

If the bending moment increases slowly, the stress at the characteristic point of the beam remains unchanged. For $t \leqslant t_1$, we have

$$\hat{\sigma}_0/E + \hat{\varepsilon}^c = \chi(\hat{z}-\delta). \tag{9}$$

Differentiating (9) with respect to time, we obtain

$$B\hat{\sigma}_0^n = \dot{\chi}(\hat{z}-\delta) = (\Delta\chi/t_1)(\hat{z}-\delta).$$

Hence, $\hat{\sigma}_0 = (\Delta\chi(\hat{z}-\delta)/(Bt_1))^{1/n}$ and $\chi_0 = \hat{\sigma}_0/(E(\hat{z}-\delta_0)) = (E(\hat{z}-\delta_0))^{-1}(\Delta\chi(\hat{z}-\delta)/(Bt_1))^{1/n}$.

At the moment t_1, the displacements of the beam are fixed and the stresses and bending moment begin to relax:

$$\dot{\hat{\sigma}}/E + B\hat{\sigma}^n = 0, \qquad t = t_1, \qquad \hat{\sigma} = \sigma_0. \tag{10}$$

Solution of (10) yields $\hat{\sigma}(t) = \hat{\sigma}_0[1 + (n-1)EB\hat{\sigma}_0^{n-1}(t-t_1)]^{-1/(n-1)}$. For $t = t_2$, we obtain $\hat{\sigma}(t_2) = \hat{\sigma}_0[1 + (n-1)EB\hat{\sigma}_0^{n-1}(t_2-t_1)]^{-1/(n-1)}$.

After the load is removed, the beam curvature decreases by the value of elastic unloading $\chi_{\text{unload}} = \hat{\sigma}(t_2)/(E(\hat{z}-\delta_0))$. As a result, it should be equal to the specified curvature of the member χ_{member}. From this it follows that

$$\chi_1 = \chi_{\text{member}} + \chi_{\text{unload}} = \chi_0 + \Delta\chi. \tag{11}$$

Substituting χ_0 and χ_{unload} expressed in terms of $\Delta\chi$ into (11), we obtain

$$\Delta\chi = \chi_{\text{member}} - \frac{1}{E(\hat{z}-\delta_0)}\Big(\frac{\Delta\chi}{Bt_1}(\hat{z}-\delta)\Big)^{1/n}\Big[1 - \Big(1 + (n-1)EB\Big(\frac{\Delta\chi_1}{Bt_1}(\hat{z}-\delta)\Big)^{(n-1)/n}(t_2-t_1)\Big)^{-1/(n-1)}\Big].$$

Solving this nonlinear equation, we obtain the quantity $\Delta\chi$, which determines all parameters of the process. Thus, given the creep characteristics of the material, geometry of the beam, and the duration of the process, one can calculate the desired residual curvature of the beam. In this case, the problem is reduced to solution of nonlinear algebraic equations. Plastic strains can easily be taken into account.

Figure 2 shows calculated curves of $\chi_1(\chi_{\text{member}})$ (the solid curve refers to a direct numerical solution and the dashed curve to a simplified calculation using the characteristic point). The calculations were performed for a beam made of AK4-1T alloy for $t_1 = 1$ h, $t_2 = 2$ h, $T = 200$°C, and $E = 60$ GPa. The beam had a T-shaped cross section and dimensions $b_1 = 1080$ mm, $b_2 = 7.5$ mm, $h_1 = 2$ mm, and $h_2 = 23$ mm. The creep constants were as follows: $n = 7$ and $B = 3.7152 \cdot 10^{-20}$ MPa$^{-n}\cdot$ sec^{-1}.

Good agreement between the direct numerical solution and the approximate solution shows that the method proposed above can be used to analyze the creep bending of beams with one symmetry axis.

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